Physics 260 Reading Notes

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Chapter 15: Mechanical Waves

Various types of physical processes involve a conveyance of energy as a wave - waves often move in sinusoidal patterns, where the motion of the particles is described by sine waves - classic example: waves traveling on a stretched string or rope - overlapping waves can also interact, resulting in interference

This chapter deals with mechanical waves, which arise as energy propagating through a medium - however, many of these concepts are applicable to electromagnetic wave that travel through empty space

15.1: Types of Mechanical Waves

Mechanical waves can arise as three types: transverse waves: the motion of the particles is perpendicular to the direction in which the wave propagates; longitudinal waves: the motion of the particles is parallel to wave propagation; and combinations, in which particles move in circular motion - in all three examples, the waves propagate through the medium with a measurable wave speed, and transport energy from one region to another; however, the medium itself does not travel

15.2: Periodic Waves

Periodic, transverse waves travel through the medium as a consistent, repeating series of crests and troughs - when a sinusoidal wave propagates through the medium, every particle experiences harmonic motion with the same frequency - angular frequency: $\omega = 2\pi f$; period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$; given a wavelength λ, wave speed: $v = \lambda f$ - however, transverse motion of the particles is not the same as the wave speed; i.e., the amplitude of the wave and the velocity of the wave are independent

In longitudinal waves, the particles move parallel to the direction of travel - the energy propagates as pressure; rather than crests and troughs, a longitudinal wave has areas of compression and areas of rarefaction - however, the same concepts apply: $v = \lambda f$

15.3: Mathematical Description of a Wave

A wave can be modeled as a wave function: $y(x,t)$ describes the motion of a particle at position x and time t - each particle moves at a different position corresponding to the phase of the wave - for a periodic transverse wave, $y(x,t) = A\cos(2\pi(\frac{x}{\lambda} \pm \frac{t}{T}))$ - note that $\pm$ depends on whether the wave is moving in the $+x$ or $-x$ direction (signs are opposite) - using wave number (number of waves per unit of length): $k = \frac{2\pi}{\lambda}$, and $y(x,t) = A\cos(kx \pm \omega t)$ - phase: $(kx \pm \omega t)$ determines the phase of the wave

The motion of each particle in the wave is further described by $v_y(x,t) = \frac{\delta y(x,t)}{\delta t}$ $= \omega A \sin(kx - \omega t)$, and $a_y(x,t) = \frac{\delta^2 y}{\delta t^2} = -\omega^2 A\cos(kx - \omega t)$ (note that each particle is always accelerating toward the x-axis)

Wave equation: $\frac{\delta^2 y(x,t)}{\delta x^2} = \frac{1}{v^2}\frac{\delta^2 y(x,t)}{\delta t^2}$ - this equation indicates that the curvature of the line at each particle $(\frac{\delta^2 y}{\delta x^2})$ is a function of its y-acceleration $(\frac{\delta^2 y}{\delta t^2})$ - this relationship is also true for longitudinal waves, except that y represents longitudinal pressure rather than transverse position

15.4: Speed of Transverse Wave

Wave speed is a function of the medium; e.g., sound waves in water propagate faster than sound waves in air - for this section, the example will be the motion of a wave in a string

The wave speed of transverse waves on a string are related to the tension of the string, t (higher tension increases the restorative transverse force on the particles, thus causing the wave to propagate faster), and its linear mass density (i.e., mass per unit length), μ (heavier particles reduce the speed of the wave)

First model: For a string stretched with tension F and linear mass density μ given an upward impulse at $t = 0$, the force on each particle (starting from the beginning of the time that it experiences the force) is $F_y t = mv_y$ - the transverse momentum is $(\mu vt)v_y$, and the wave speed is $v = \sqrt{(\frac{F}{\mu})}$ - the wave speed in this model is not dependent on v_y, a_y, f, or T, but is solely based on the tension and the linear mass density

Second model: Consider the force on each segment - the mass of the segment is $m = \mu\delta x$ - the longitudinal force on each end of the segment is related to its position along the segment of string: $\frac{F_{1y}}{F} = -(\frac{\delta y}{\delta x})_x, \frac{F_{2y}}{F} = (\frac{\delta y}{\delta x})_{x+\delta x}$ - taking the limit as $\delta x \to 0$: $\frac{\delta^2 y}{\delta x^2} = \frac{\mu}{F}\frac{\delta^2 y}{\delta t^2}$ - comparing this with the wave equation, $v = \sqrt{\frac{F}{\mu}}$

These observations describe a variety of conditions: $v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$

15.5: Energy in Wave Motion

For a wave traveling left-to-right along a string, the instantaneous power delivered along the string (i.e., the power delivered by each point to each connected, following point) is $P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\delta y(x,t)}{\delta x}\frac{\delta y(x,t)}{\delta t}$ - for a sinusoidal wave, $P(x,t) = Fk\omega A^2\sin^2(kx - \omega t) = \sqrt{\mu F}\omega^2 A^2\sin^2(kx - \omega t)$, and the average power is half of the maximum power (at $\sin^2(kx - \omega t) = 1$): $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

For all types of mechanical waves, the average rate of energy transfer is proportional to (frequency $\cdot$ amplitude)^2 - however, for electromagnetic waves, the average rate of energy transfer is only proportional to amplitude^2, and is independent of frequency

Wave intensity: For waves that travel in multiple dimensions (e.g., ripples on a pond), the intensity is defined as the rate at which energy is transported along each unit area of a surface oriented perpendicular to the direction of propagation (e.g., for each unit length along a curved sheet inserted into the surface of the pond) - for a three-dimensional sphere, the average intensity $I = \frac{P}{4\pi r^2}$ - also, for two spheres with different radii r_1, r_2, the ratio of the intensity is relative to the squares of the radii: $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$

15.6: Wave Interference, Boundary Conditions, and Superposition

Reflection: If a string is fixed to a point, a wave propagating along the string will impart its energy into the fixed point, which then rebounds the energy back into the string - as a result, the wave is reflected back along the string: same amplitude, but opposite direction of displacement - conversely, a string that is attached to a point that allows transverse motion (e.g., a ring sliding along a frictionless rod) causes the wave to be reflected, but with the same direction of displacement - thus, the reflection depends on the boundary condition of the string

Interference: When two waves encounter each other in a typical medium, the result is the superposition of the waves, i.e., the algebraic sum of the displacements: $y(x,t) = y_1(x,t) + y_2(x,t)$

15.7: Standing Waves on a String

A periodic wave on a string with a fixed end causes recurring interactions between the incident waves and the reflected waves - depending on the frequency, this pattern can result in "standing waves" that appears to move transversely but not longitudinally - actually, each standing wave is a superposition of two interacting traveling waves - each standing wave spans a pair of nodes (non-moving points) and features an antinode (point where the amplitude is greatest) - the standing wave also ranges from a moment of destructive interference (smaller amplitude than each wave), with a minimum at complete destructive interference (flat string), to a moment of constructive interference (greater amplitude than each wave), with a maximum at complete constructive interference

Wave equation for standing waves: $y(x,t) = (A_{\text{SW}} \sin(kx) \sin(\omega t)$, where A_{SW} = 2A (i.e., twice the amplitude of each individual wave) - in contrast with a standing wave, all points along the string are in the same phase at all times (i.e., every point along the string is flat, and at max amplitude, at the same moment in the cycle) - each point is the superposition of $y_1(x,t) = -A\cos(kx+\omega t)$ and $y_2(x,t) = A\cos(kx+\omega t)$ - the nodes are at $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$ - a standing wave transfers no power: $P_{AV} = 0$

15.8: Normal Modes of a String

For a wave of length L that is fixed at both ends, a standing wave can develop with one node at each end: $\lambda_n = \frac{2L}{n}(n = 1, 2, 3, \ldots)$ - waves with other wavelengths will not result in a standing wave - since $f_n = \frac{v}{\lambda_n}$, the frequencies that produce standing waves are $f_n = n\frac{v}{2L}(n = 1, 2, 3, \ldots)$ ("harmonic" frequencies), which include the smallest wave frequency that will produce a standing wave ($f_1 = \frac{v}{2L}$) ("fundamental frequency"), and higher frequencies or "overtones" - combining this with the examples of string tension: for a string with a fixed length / tension / length mass density, $f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$ (describing both the fundamental frequency of the string, and of sound waves created by the string vibration)

For any oscillating system, the "normal mode" of the system is the scenario where all particles move sinusoidally with the same frequency - while many of the examples presented in this chapter presume that the shapes of the waves produce simple patterns, more complex patterns can be achieved through interference of waves of different shapes, resembling the superposition of many different normal modes - many oscillating systems present various types of "harmonic content" caused by different types of wave events (e.g., plucking and hammering a guitar string produce waves with different shapes that result in various types of frequencies higher than the fundamental, i.e., different types of harmonic content)

Chapter 16: Sound and Hearing

16.1: Sound Waves

Sound is any longitudinal wave in a medium (typically air, but can be any gas, liquid, or solid) - simple sound waves are just sinusoids, which are audible within the range of 20 to 20,000 Hz as a tone (bordered by infrasonic and ultrasonic waves) - typically travel outward in all directions from source of sound - a sinusoidal sound wave propagates according to the wave equation $y(x,t) = A\cos(kx - \omega t)$; this motion represents the displacement amplitude of each particle in the medium

As with all longitudinal waves, sound waves travel as fluctuations in longitudinal pressure - microphones and eardrums detect pressure fluctuation (as compared with atmospheric pressure) rather than particle displacement - the pressure at each point is $p(x,t) = BkA\sin(kx - \omega t)$, where B is the bulk modulus: $p(x,t) = -B\frac{\delta y(x,t)}{\delta x}$ - the maximum pressure $p_{max} = BkA$

The frequency of the sound wave determines its pitch - loudness is generally proportional to amplitude of displacement, but is subjective, in part because the eardrum is not equally sensitive to all frequencies - e.g., aging causes a gradual loss of sensitivity at the high end of the spectrum

Musical instruments and human voices typically produce sound with a distinctive variety of frequencies, giving rise to a characteristic harmonic content that is perceived as the timbre of the instrument or voice ("reedy," "mellow," "tinny," etc.) - other characteristics of sound: attack (rapidity of onset), decay (rapidity of fading) - noise is a combination of an unresolvable combination of many frequencies; "white noise" is the combination of all frequencies

16.2: Speed of Sound Waves

For the example of a pipe of cross-sectional area A, filled with a fluid with a pressure p, the longitudinal velocity of the wave is $v = \sqrt{\frac{B}{\rho}}$, where B is the bulk modulus of the fluid, and ρ is the mass per unit volume - similarly, for a solid with sides that are free to reverberate, the speed is $v = \sqrt{\frac{Y}{\rho}}$, where Y is Young's modulus (i.e., tensile modulus or elastic modulus) - however, for bulk solids where the sides are confined and cannot move, the speed of sound is also related to the shear modulus

Sound waves traveling in fluids are used by dolphins for echolocation, and in medical ultrasound imaging; high frequencies and short wavelengths enable high-resolution detection of reflected surfaces and propagating mediums, e.g., tissue density

The speed of sound waves in a gas is more complicated, as the pressure of the gas can more readily fluctuate, and because the bulk modulus of gas varies proportionally with the current pressure: $B = \gamma p_0$, where γ is the ratio of heat capacities for the gas, and p_0 is the equilibrium pressure of the gas - the speed of a sound wave in gas is $v = \sqrt{\frac{\gamma RT}{M}}$, where R is the gas constant (R = 8.314472 J / mol K), T is the temperature, and M is the mass per mole of the gas

16.3: Sound Intensity

As with mechanical waves, the energy conveyed by a sound wave can be measured as wave intensity, defined as the average rate of energy is transported per unit area across a surface positioned perpendicular to the direction of propagation - for sinusoidal sound waves, $I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$

The maximum pressure amplitude is a function of the sound intensity: $I = \frac{p_{max}^2}{2\sqrt{\rho B}} = \frac{p_{max}^2}{2\rho v}$ -

Sound waves typically propagate in all directions equally, and thus intensity dissipated proportional to $\frac{1}{r^2}$ - however, reflections and directed sound emission can focus the energy in a desired direction, and can reduce the rate of dissipation

Decibel scale: $= (10\text{dB}) \log \frac{I}{I_0}$ - the logarithmic scale suggests that every increase in 10 dB requires a 10-fold increase in intensity, with 120 dB = 1 W/m$^2 asabaseline, and 0 setatthethresholdofhearingat 1000Hz, andanintensityof 10^{-12}$ W/m^2

16.4: Standing Sound Waves and Normal Modes

Standing waves can arise for longitudinal sound waves just like transverse mechanical waves - as an example, a tube having a diaphragm on one end and a rigid, flat (i.e., reflective) surface at the other end is filled with a medium, and the diaphragm is vibrated at a selectable frequency using a speaker - at normal modes, a standing wave is created with nodes (locations of no displacement) and antinodes (locations of maximum displacement) - at the nodes, the particles on either side are exactly out of phase, and the location at the node experiences the maximum amount of compression and rarefaction; at the antinodes, the particles on either side are exactly in phase, causing maximum displacement but minimum pressure fluctuations

In the example of a tube, the boundary behavior of longitudinal sound waves mirrors that of transverse waves - if the end is closed, the wave is reflected with the same shape but opposite phase (i.e., the point at the end is a node); if the end is open, the wave is reflected with the same shape and identical phase (i.e., the point at the end is an antinode)

An organ pipe can be designed with both ends open (an "open pipe") or with one end closed (a "stopped pipe") - an open pipe operates as if it has an antinode at each end and a node in the middle, with a fundamental frequency $f_1 = \frac{v}{2L}$ with $= 2L$, and operate at higher harmonics according to $f_n = \frac{nv}{2L} = nf_1, n = (1, 2, 3, \ldots)$ - for a stopped pipe, the open end is an antinode, but the closed end is a node, and $f_1 = \frac{v}{4L}$, and higher harmonics are represented as $f_n = \frac{nv}{4L} = nf_1, n = (1, 3, 5, \ldots)$ - i.e., for the harmonics $\frac{nv}{4L}$, the odd-numbered harmonics arise in closed pipes, and the even-numbered harmonics arise in open pipes

As with sound produced by transverse waves, sound produced by instruments using longitudinal waves (e.g., organs and flutes) is composed of distinctive harmonic content that produces a characteristics timbre - while organ pipes only produce one pitch, other wind instruments (e.g., flutes) have holes that allow variable lengths that produce different pitches - however, because pressure is a function of temperature, instruments produce higher-pitch notes at higher temperatures

16.5: Resonance and Sound

Oscillations inside a tube can be "driven" by a source of oscillating pressure (e.g., a speaker positioned next to a diaphragm cap of a tube) - for most "driving" frequencies (i.e., the frequency of the oscillating force), the pressure waves caused by each oscillation within the tube will mostly interfere destructively; however, at the normal modes of the tube, the pressure waves interfere constructively, resulting in a maximization of the wave amplitude

16.6: Interference of Waves

Overlapping waves can interfere in various ways, including producing standing waves and resonance - as another example, consider two speakers generate sound that is exactly in phase - at many locations in the room (particularly locations that are equidistant from the speakers), the sound waves will arrive perfectly in phase and will constructively interfere, producing sound twice as loud as each speaker - at other locations in the room, the arriving sounds are partially or wholly out of phase, and will have much smaller amplitude (this situation resembles a standing wave, but is actually a traveling wave) - more specifically, constructive interference occurs when the location is a whole wavelength closer to one speaker than the other, and destructive interference occurs when the location is a half-wavelength closer to one speaker than the other

16.7: Beats

Interference among sound waves with slightly differing phases can give rise to a "beat" note with an amplitude that oscillates between zero and a positive value - the rate of oscillation is the "beat frequency," and is the difference between the frequencies of the sound waves - at larger differing frequencies, the sound waves merge into a difference tone, which can be consonant or dissonant (depending on the frequency ratios)

16.8: The Doppler Effect

When a sound source and a sound listener are stationary, sound frequencies are as described above; however, when either or both of the source and listener are moving relative to the medium, the frequency of the detected sound is shifted - when the source and listener are moving closer, the sound wave is detected at a higher frequency, and when the source and listener are moving farther apart, the sound wave is detected at a lower frequency - specifically, $f_L = \frac{v+v_L}{v+v_S} f_S$ - note: this equation depends on the motion of each of the source and listener with respect to the medium, not just their relative velocities

Doppler shift for electromagnetic waves: When calculating Doppler effects for sound traveling through a medium, v_L and v_S are measured with respect to the medium - for electromagnetic waves traveling through space, Doppler effects are measured relative to the speed of light: $f_R = \sqrt{\frac{c-v}{c+v}} f_S$, where v is the relative velocity of the source and receiver - this can be used to detect object velocities, e.g., in a radar detector

16.9: Shock Waves

When an object moves through a medium near the speed of sound, the sound waves build up in front of the object, creating significant pressure that imposes extra aerodynamic drag on the object, and that the object must push through to accelerate further - at supersonic speeds, the sound waves interfere constructively to create shock waves, like the wake of a boat, that radiate outward from the object in the shape of a cone that moves with the velocity of the object - the angle of the cone is $\sin(\alpha) = \frac{v}{v_S}$

Chapter 21: Electric Charge and Electric Field

Electromagnetism is one of the four fundamental forces - electromagnetic interactions involve particles having an electric charge, which is quantized and conserved - charges at rest exert an electrostatic force on one another, based on a simple relationship (Coulomb's law), and create an electric field - later chapters focus on charges in motion

21.1: Electric Charge

Electric charge: originally described as the property exerted by rubbing amber with wool, a glass or plastic rod with silk, or a comb through hair - electrostatics: the interactions between charges that are at rest - charges have a sign (positive and negative); like charges attract, and opposite charges repel - rubbing a glass rod with silk imparts a positive charge on the rod, and rubbing a plastic rod with silk imparts a negative charge on the rod

Electric charges arise from protons and electrons (charges have identical magnitude in both particles, but opposite sign) - protons are mostly confined to the nucleus (and overcome the repulsion of like charges by the strong nuclear force); electrons can move to redistribute charge - an atom having equal protons and neutrons has no charge, but can be ionized to add an electron (inducing negative charge) or to remove an electron (inducing positive charge) - the charge on an electron or proton is the natural quantum of charge (though quarks can have fractional charges)

Conservation: The sum of all electric charges in any closed system is constant

21.2: Conductors, Insulators, and Induced Charges

Conductive materials (particularly metals) allow charge to move from one region of material to another; insulators resist movement of charge; semiconductors are intermediate - atoms of conductive elements allow some electrons (e.g., one) to move to other atoms, while other elements remain bound to atom

Electric forces can affect even uncharged objects: proximity creates a dipole, and the opposite sides attract

Conductive materials can have an imparted charge - e.g., charging a plastic rod (thus inducing a negative charge) and touching it to a metal ball transfers the negative charge the the ball, which will now attract a positively charged glass rod - charge can be dissipated by connecting it to the earth (as an infinitely large conductor of charge) - Figure 21.7: grounding can result in a net positive electrostatic charge: position a negative charge near a metal ball (inducing a dipole), connect a wire from the opposite side to ground (dissipating the negative charge), and then remove the negative charge; result is a net positive charge on the ball

21.3: Coulomb's Law

Charles Augustin de Coulomb found that point charges exert a force on each other: $F = k\frac{|q_1 q_2|}{r^2}$ (k is a proportionality constant) - even when charges have different magnitudes, the force felt on each is equal

Proportionality constant can be expressed in various ways:

- SI unit of charge is one coulomb (1 C), and proportionality constant $k = 8.988x10^9 \frac{N \cdot m^2}{C^2}$ - the value is related to the speed of light in a vacuum ($k = 10^{-7} \frac{N \cdot s^2}{C^2} \cdot c^2$) - note that one coulomb is a very large charge; real-world problems typically involve charges in the range of microcoulombs or nanocoulombs
- The coulomb can be defined as a unit of electric current: 1 ampere = 1 coulomb / second
- The coulomb can also be expressed in terms of the charge of an electron (e): $C =$ the total charge of $6x10^{18}$ electrons, and $e = 1.602x10^{-19}C$
- The coulomb can also be written in terms of another constant $\epsilon_0 = 8.854x10^{-12}C^2 \cdot m^2$, i.e., $k = \frac{1}{4\pi\epsilon_0}$, so that $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$

Superposition of forces: When two charges exert forces on a third charge, the total force on the third charge is the sum of the force vectors exerted by the individual charges (note: this applies in a vacuum or air, this is acceptable, but can change if other materials are present)

21.4: Electric Field and Electric Forces

Coulomb's law can be reformulated as an electric field: consider that the first particle changes space around it, a second particle positioned somewhere in the space exhibits the effects of the field - the strength and direction of the effect on the second particle at a point in space around the first particle is a function of the electric field at that point created by the second particle

The electric force on a charged body is exerted by the electric field created by other charged bodies - i.e., $\overrightarrow{E} = \frac{\overrightarrow{F_0}}{q_0}$ (electric field = electric force per unit charge) - alternatively, $\overrightarrow{F_0} = q_0 \overrightarrow{E}$ - as per Coulomb's law, the second particle also creates an electric field, resulting in an equal and opposite force on the first particle - note: the electric field equation can only be used for a point test charge - thus, the force on a particular point having charge q_0, and at a distance r from a point charge q: $F_0 = \frac{1}{4\pi\epsilon_0} \frac{|qq_0|}{r^2}$ - in terms of a unit vector $\hat{r}$, $\overrightarrow{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

Electric field can be illustrated as a vector field - by convention, the electric field of a point charge radiates away from a positive charge, and toward a negative charge - a uniform field has the same magnitude and direction of charge everywhere - example: conductors exert an electric field on charged particles - in electrostatics, the electric field at every point within the material of a conductor must be zero

21.5: Electric-Field Calculations

Superposition of electric fields: The electric field exerted on a particle at a position in space is the sum of the force vector exerted by each other particle in the space - charge can be distributed across a line (linear charge density, measured in $\frac{C}{m}$), over a surface (surface charge density, measured in $\frac{C}{m^2}$), or within a volume (volume charge density, measured in $\frac{C}{m^3}$) - the rest of this section presents some example calculations

21.6: Electric Field Lines

Electric fields can be illustrated as an electric field map: a set of streamlines illustrating the direction of the electric field at each point (direction = line tangent), and the density of the streamlines approximately illustrates the magnitude of the field - since field lines must be unambiguous, the streamlines can never intersect - in accordance with convention, field lines radiate from positive charges and converge on negative charges - for a uniform field, the streamlines are straight, parallel, and uniformly spaced

21.7: Electric Dipoles

A dipole is a pair of point charges with equal magnitude and opposite sign separated by a distance d - the water molecule is an electric dipole: hydrogen atoms exhibit a small positive charge, and the oxygen atom expresses a small negative charge; this makes water a great solvent for ions such as salt field c In a uniform electric field, the net force on a dipole is zero - however, a force is exerted on each charge in the dipole, leading to a torque based on the center of the bond: $\tau = (qE)(d\sin(\phi))$, where $d\sin(\phi)$ is the perpendicular distance between the lines of action of the two forces - the quantity qd is called the electric dipole moment: usually represented simply by magnitude p, but can also be represented as a vector $\overrightarrow{p}$ factoring in the direction of the electric field (positive dipole is in the direction of the field; negative dipole is in the direction opposite the field) - note that the magnitude of the dipole moment is normalized for a particular dipole: $\tau = pE\sin(\phi)$, and $\overrightarrow{\tau} = \overrightarrow{(}p)x\overrightarrow{E}$

Potential energy of dipole: A dipole in an electric field changes direction, which exerts work done on the dipole by the torque and the displacement - the work done at each point $= dW = \tau\, d\phi = -pE\sin(\phi)\, d\phi$, and the total work done to displace from ϕ_1 to ϕ_2 is $W = \int_{\phi_1)^{\phi_2}(-pE\sin(\phi))\,d\phi = pE(\cos(\phi_2) - \cos(\phi_1))}$ - the work done represents the change in potential energy, and the dipole orients to reduce potential energy

In a nonuniform electric field, a dipole can experience a net force - i.e., an uncharged body can be polarized by an electric field, thus inducing an electric dipole moment, which is how uncharged bodies respond to electrostatic forces -

dipoles also create electric fields, but the field calculations become very complicated; best evaluated by computer simulation

Chapter 22: Gauss's Law

Gauss's Law is a conceptual model that simplified electric field calculations, particularly of the distribution of charge over a surface - in general, Gauss's Law notes that a surface enclosing a charge exhibits a flux magnitude that is irrespective of the shape and size of the surface - thus, rather than calculating flux over an irregularly shaped surface, we can devise a regular, symmetric surface over which enables easier calculation

22.1: Charge and Electric Flux

Chapter 21 dealt with computing the electric field $\overrightarrow{E}$ at a point P as the superposition of the fields created by other points - we can also ask about the charge distribution over a region, such as within a closed surface such as a sphere, or an open surface such as a plane - the electric flux represents the field over a surface - by convention, positive electric flux = electric field directed through the surface form a convex side to a concave side, or from inside to outside for a closed surface; negative electric field = opposite direction - flux is always measured as the component normal to the surface: a flat surface parallel to a uniform field encounters a positive E-field, but has zero flux

Electric flux properties:

- For a charge inside a closed surface, the electric flux is directly proportional to the magnitude of the charge - this is independent of the size and shape of the surface - e.g., a larger surface has a weaker flux at each point (decreases by $\frac{1}{r^2}$, but the surface area increases, and these properties offset each other
- For a closed surface, a charge outside the surface induces a net zero flux: the electric flux at each point flowing into the surface is offset by the electric flux flowing out of the surface at another point

- For a closed surface, the electric flux is directly proportional to the magnitude of the charge, and is a superposition of the magnitudes of several charges

22.2: Calculating Electric Flux

The calculation of electric flux resembles the calculation of fluid flow; e.g., both depend on the angle of the surface and the direction of the field

For a flat surface, electric flux $\Phi_E = EA\cos(\phi)$, where E = strength of field and A represents the surface area of the surface, and ϕ represents the angle between a vector normal to the surface and the direction of the electric field - for a uniform field of strength E directly penetrating a surface with area A, $\Phi_E = EA$; for a uniform field parallel to a flat surface, $\Phi_E = 0$

More generally, for any surface, $\Phi_E = \int \overrightarrow{E} \cdot d\overrightarrow{A}$ (this is the surface integral of the perpendicular component of the electric field) - the magnitude is an average of the E field across the surface; local variations may occur

22.3: Gauss's Law

Gauss's Law: Total electric flux through any closed surface is proportional to the net electric charge inside the surface: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$ - note that $\overrightarrow{E}$ is normal to the surface at every point

General form of Gauss's Law: $\Phi_E = \oint \overrightarrow{E} \cdot d\overrightarrow{D} = \frac{Q_{encl}}{\epsilon_0}$ - this is true for any surface, since the total flux through any closed shape must be the same as for a sphere - thus, when evaluating any surface, we can choose any imaginary Gaussian surface for the same electric flux calculation - can also be expressed as $\Phi_E = \oint E\cos(\phi)dA$

For a Gaussian sphere with a charge at the center, electric flux is easy to calculate, since the electric field vector is normal and of identical magnitude for every point - q can be taken out of the integral, and the surface integral is just $4\pi r^2$, resulting in $\Phi_E = \frac{q}{4\pi\epsilon_0 r^2}4\pi r^2 = \frac{q}{\epsilon_0}$

22.4: Applications of Gauss's Law

Gauss's law can answer two (inverse) questions: (1) given an electric field map, calculate the charge distribution within a region; and (2) given the charge distribution within a region, identify the electric field map

This subsection presents some exemplary calculations - one fact that facilitates calculations: when excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior (if the electric field inside the conductor weren't zero, the charge inside the conductor would move)

22.5: Charges on Conductors

As noted in 22.4, in an electrostatic scenario, a conductor has no charge on its interior; any exhibited electrostatic charge is on the exterior - if the conductor has an interior cavity containing no charge, then the interior surface of the conductor forming the cavity also has a net charge of zero (since an arbitrary Gaussian surface formed within the conductor material contains no charge, and hence has an electric flux of zero) - additionally (proven later), the interior surface exhibits zero charge at every point - alternatively, if the cavity contains a net charge of one sign, then the interior surface of the cavity must have a net charge of the opposite sign in order to negate the cavity charge and maintain the electrostatic scenario

Faraday ice-pail experiment: Charging a metal ball and suspending it within a conducting container causes the electrostatic situation described above, where the surface of the container exhibits a dipole, with the interior of the container having one charge negating the charge of the metal ball and the exterior of the container having the equal and opposite charge - touching the metal ball to the side of the container causes it to become part of the cavity; the ball and the charge of the interior of the container are negated, and all that remains is the charge on the exterior of the container (equalling the magnitude of the charge of the ball)

Electrostatic shielding: Forming a hollow box with a conductive metal surface ensures that electric charges outside of the box remain on the surface, and not

not affect the interior or contents of the box

For any conductor, the surface charge density σ at a point is related to the electric field at that point: the field is normal to the surface, and the magnitude $E = \frac{\sigma}{\epsilon_0}$

Chapter 23: Electric Potential

When an electric field exerts a force on a particle that results in movement, the electric field is exerting work on the particle - the ability of an electric field to perform work on particles is termed the electric potential of the field (in electric circuits, electric potential is called voltage)

23.1: Electric Potential Energy

In mechanical systems, work is defined as $W_{a\to b} = \int_b^b \overrightarrow{F} \cdot d\overrightarrow{l} = int_a^b F\cos(\phi)\, dl$, where $d\overrightarrow{l}$ = displacement along particles path and ϕ = angle between $\overrightarrow{F}$ and $d\overrightarrow{l}$ - if force is conservative, the amount of work that a force can achieve is called the potential energy, or U, and the work performed by moving between two points equals the change in potential: $W_{a\to b} = U_a - U_b = -\Delta U$

These concepts translate directly to electric potential: $W_{a\to b} = Fd = q_0 Ed$ (i.e., the work required to move a charged particle over a distance = the force times the distance, or the charge of the particle, the magnitude of the electric field, and the distance) - electric force is conservative: change in potential energy is inverse to the work performed - accordingly, the potential energy at a point $U = q_0 Ey$ (where y is the distance between the particles) - of course, unlike gravity, electric fields can also repulse; in general, moving in the same direction of the electric field does positive work on the particle and reduces potential energy, and moving in the opposite direction of the electric field does negative work on the particle and increases potential energy

These equations are easy in uniform electric fields where E is a constant - in dynamic electric fields, the calculations have to consider changing magnitude and changing force: $W_{a\to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\, dr$ - and for movement of the particle that is not always in the direction of the electric field, $W_{a\to b} = \int_{r_a}^{r_b} F\cos(\phi)\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\cos(\phi)\, dl$, and the potential energy at any point $U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$ - the signs of the charges don't matter to the actual magnitude of the force

Potential energy is relative to a reference state where potential energy is zero; for electrical potential, the reference state is where two particles are at infinite distance - also, for two particles, the potential energy U of each on the other is a shared property that is identical for both particles (though the total potential energy for each particle may also be affected by other particles) - also, the potential energy of a uniformly charged sphere with total charge q on a particle is identical to the potential effect of a point charge q on the same particle

For a set of charges, the total potential energy U of a selected particle is the sum of the potential energies with each other particle: $U = \frac{q_0}{4\pi\epsilon_0}(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots)$ - note: not a vector sum, but a quantity sum - conceptual model: think about having all particles at infinite distance, and then "assembling" them by positioning each one in the field; the potential energy is opposite the amount of work require to put the particle in place

23.2: Electric Potential

Potential energy is a measure of work achievable by a particular particle - "potential" = potential energy per unit charge: $\frac{V=U}{q_0}$ (alternatively, potential energy = potential times charge: $U = Vq_0$) - standard unit of potential is the volt: 1 joule / coulomb - accordingly, $\frac{W_{a\to b}}{q_0} = V_a - V_b$ - the potential for a first point charge with respect to a second point charge (with charge q and at distance r: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ - and for a set of charges: $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$ - when charge is distributed over a line, $V = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r}\, dq$

It's also possible to find the electric potential from the electric field: $V_a - V_b = \int_a^b \overrightarrow{E} \cdot d\overrightarrow{l} = \int_a^b E\cos(\phi)\, dl$ - as with potential energy, a positive result indicates that positive work is being done on the particle and the potential energy decreases, and a negative result indicates that negative work is being done on the particle and potential energy increases

Units of measurement: Since one volt = 1 joule / coulomb, the energy required to move a particle over a distance can be measured in volts or in newtons per coulomb: $1V/m = 1N/c$ - we can also consider the amount of energy required to move a single electron, i.e., an electron-volt (eV): 1 eV = 1.602×10^{-10} joules - this can be used as a unit of measure of potential energy change between two points: $U_a - U_b = (1.602 \times 10^{-19}C)(1V) = 1.602t \times 10^{-19}J$ - the electron-volt is simply a unit of measurement of energy (including change in potential between two points), while the volt is a unit of measurement of potential

23.3: Calculating Electric Potential

Electric potential can be calculated by two methods: (1) from a known charge distribution, or (2) from the electric field and the work required to move through the electric field from a particular point (e.g., an infinite distance, where electric potential is zero)

For a conductor surrounded by a material such as air, the maximum charge with which the conductor can be charged is limited by the tendency of the surrounding material to become ionized and conduct charge away from the conductor - e.g., air becomes ionized at $3x10^6 \frac{V}{m}$, so the surface charge of any conductor cannot result in a surface charge density greater than this; for a sphere with radius $R, V_{max} = RE_{max} = R \cdot 3x10^6$

23.4: Equipotential Surfaces

In an electric field diagram, lines can be drawn indicating the path within space of equal electric potential - the lines can form equipotential surfaces, and particles can move freely along an equipotential surface without experiencing any change in potential energy - like electric field lines, equipotential surfaces can never intersect; and equipotential surfaces are always perpendicular to electric field lines - equipotential surfaces resemble a contour plot: areas with a more dense illustration of equipotential surfaces are areas where the magnitude of the electric field is large, such that large changes in electric potential occur over short distances

In electrostatics:

- The surface of a conductor is always an equipotential surface
- The electric field just outside the conductor must be an equipotential surface
- The potential within the body of a conductor is at the same potential (if it were not, charges would move)

23.5: Potential Gradient

The potential gradient indicates the direction at which the potential is reduced most strongly for a point in space - the potential gradient is always in the opposite direction as the electric field: $\overrightarrow{E} = -\overrightarrow{\Delta} V$ - the gradient, like the electric field, is always perpendicular to an equipotential surface - this provides another way to find the electric field, as the opposite of the gradient of the potential: $E_r = -\frac{\partial V}{\partial r}$

Chapter 24: Capacitance and Dielectrics

Capacitors store electric charge and electric potential - simple example: two conductors insulated from each other, with charge transferred from one conductor to the other to create polarization - reconnecting the conductors causes a flow of electric charge and a release of electric potential - the ratio of the charge difference is called the capacitance of the capacitor, and is a factor of the size and shape of the conductors and the insulation - a vacuum is one option, but higher capacitance is achievable by including a dielectric that insulates the conductors - a key idea is that the potential energy is considered to be stored by the electric field

24.1: Capacitors and Capacitance

A capacitor comprising a pair of polarized conductors stores a charge Q - one conductor has charge $+Q$ and is at higher potential; the other has charge $-Q$ and is at lower potential - when connected to and charged by a battery with voltage V_{ab}, the capacitor is charged with a potential difference V_{ab} - both the resulting electric field in the region between the conductors and the potential difference V_{ab} are relative to the magnitude Q of the capacitor charge: the capacitance $C = \frac{q}{V_{ab}}$, and the standard of capacitance is the Farad $= \frac{\text{1 coulomb}}{\text{1 volt}}$, or $\frac{\text{1 Coulomb}^2}{\text{1 Joule}}$ - capacitance measures the magnitude of the charge induced by a particular voltage, and is a product of the shape and sizes of the conductors and the insulator

Simple example: A parallel-plate capacitor features two plates of area A separated by a vacuum of distance d that is small compared with A - the field between the plates is comparatively uniform, and the charge is uniformly distributed over each plate - the field magnitude $E = \frac{Q}{\epsilon_0 A}$, and the voltage difference $V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}$ - thus, the capacitance $C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$ - the capacitance is independent of the charges on the plates or the potential difference

Note that ϵ_0 can be expressed as $\frac{8.85x10^{-12}F}{m}$ - this is useful in capacitance calculations

24.2: Capacitors in Series and Parallel

Capacitors can be connected in series on a wire between points a and b - if both are uncharged and exposed to a constant positive potential difference V_{ab}, the capacitors are all charged to the same magnitude Q - this is because the positive charge on the first plate of the first capacitor comes from pushing charge to the second plate, which in turn pulls charge from the first plate of the second conductor, etc. - as a result, the system only reaches equilibrium when the magnitude of charge is equal between each consecutive pair of plates (e.g., either the plates of one capacitor, or the second plate of one capacitor and the first plate of the next capacitor in series) - the total charge held: $\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$, and the equivalent capacitance $C_{eq} = \frac{Q}{V}$, and $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ - note that (1) the total capacitance is less than any individual capacitance, and (2) the magnitude of charge Q on each capacitor in series is identical, but the capacitance and potential V of each capacitor can differ

When connected in parallel on a wire between points a and b, the individual capacitors exhibit an identical potential V, although the individual capacitance and quantity of charge can differ - the total charge $Q = Q_1 + Q_2 + \ldots = (C_1 + C_2 + \ldots)V$ - i.e., the set of high-potential plates forms a first equipotential surface, and the set of low-potential plates forms a second equipotential surface - together, the capacitors exhibit a charge equal to the sum of the charges on each plate ($C_{eq} = C_1 + C_2 + \ldots$)

24.3: Energy Storage in Capacitors and Electric-Field Energy

Capacitors can be considered a store of energy (i.e., the energy required to charge the plates to a polarity, and the energy released during discharge) - the total energy $V = \frac{Q}{C}$, and the potential energy is $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$ - the work required to charge it is the same as the potential energy stored: $W = \frac{Q^2}{2C}$

A conceptual model: charging a capacitor from 0 to a particular charge Q is initially easy but becomes more difficult, because charge is moving against the growing electric field - thus, the energy can be considered stored in the field between the plates, and calculated per unit volume of space: energy density $u = \frac{\frac{1}{2}CV^2}{Ad}$, or $u = \frac{1}{2}\epsilon_0 E^2$

24.4: Dielectrics

Any capacitor can be charged to a magnitude Q that causes partial ionization of the insulator, resulting in a flow of charge and a loss of capacitance - this is called dielectric breakdown, and can be prevented by using an insulating material (dielectric) capable of withstanding a very large charge before ionizing (e.g., Mylar) - using a good dielectric increases both the maximum charge of the capacitor, and the capacitance: greater degrees of insulation cause a weaker electric field to be created between the plates for a given charge, and thus enable less work to load the plates with the given charge - i.e., because $C = \frac{Q}{V}$, for a given charge Q, a higher capacitance C enables loading the charge with less work (lower V), but also releases less potential energy when discharged

The dielectric constant K of a material is the ratio of the original capacitance of the capacitor and the new capacitance: $K = \frac{C}{C_0}$; for a fixed charge Q, $V = \frac{V_0}{K}$, i.e., higher dielectric constants reduce the amount of work required to load the capacitor with a specific amount of charge - (in reality, no dielectric is a perfect insulator, and there is always some leakage current between the charged plates despite a capacitor, but we ignore this for a basic discussion of capacitance)

The dielectric constant also affects the strength of the electric field between the plates: $E = \frac{E_0}{K}$ (for a constant Q) - however, each plate is still a conductor with the charge Q uniformly distributed over its surface (with charge distribution σ) - the dielectric actually becomes polarized: each surface of the insulator exhibits an induced charge that partially negates the electric field exhibited by each conducting plate, with charge density σ_i - thus, $\frac{E=\{\sigma-\sigma_i}{\epsilon_0}$ - the induced surface charge density of the dielectric is a function of the surface density of the capacitor and the dielectric constant: $\sigma_i = \sigma(1 - \frac{1}{K})$ - the permittivity of the dielectric is $\epsilon = K\epsilon_0$, and $E = \frac{\sigma}{\epsilon}$ - accordingly, the capacitance $C = K\epsilon_0 \frac{A}{d} = \frac{\epsilon A}{d}$, and the energy density of the electric field $u = \frac{1}{2}\epsilon E^2$

24.5: Molecular Model of Induced Charge

Induced surface charge on a dielectric is not achieved by conduction, since an insulator cannot be conductive - rather, the induced surface charge is achieved by molecular reorientation to exhibit a polarity - even molecules that are not inherently polar, like water, can exhibit an induced polarity as charges realign within the molecule - the induced polarity is typically not enough to negate the electric field, but significantly reduces it by a certain ratio (i.e., by the dielectric constant)

24.6: Gauss's Law in Dielectrics

For a Gaussian surface enclosing a portion of a conductor and an equivalent portion of a dielectric, the flux is a function of the enclosed charge, which is the difference between the surface charge of the enclosed portion of the conductor and the induced charge of the insulator (which is less, leading to a nonzero contained charge) - since the induced charge of the dielectric is expressed by the dielectric constant, a modified version of Gauss's law is $\int K\overrightarrow{E}\cdot d\overrightarrow{A} = \frac{Q_{encl-free}}{\epsilon_0}$, where $Q_{encl-free}$ is the total free charge enclosed by the Gaussian surface, i.e., the amount of charge on the surface of the conductor (not including the bound induced charge of the insulator)

Important Formulae

- Capacitance: $C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$; $Q = VC$
- Capacitor electric field: $E = \frac{V}{d} = \frac{Q}{A\epsilon_0}$
- Spherical capacitor (plates with radius r_1, r_2): $C = 4\pi\epsilon_0 \frac{r_1 r_2}{r_2 - r_1}$
- Capacitors in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ (constant charge, varying voltage drops)
- Capacitors in parallel: $C_{eq} = C_1 + C_2 + \ldots$ (constant voltage, varying charge)
- Energy required to charge capacitor: $U = \frac{Q^2}{2C} = \frac{1}{2}QV$
- Energy density of capacitor: $u = \frac{1}{2}\epsilon_0 E^2$
- Dielectric: $K = \frac{C}{C_0}$; $C = K\epsilon_0 \frac{A}{d}$; $u = \frac{1}{2}K\epsilon_0 E^2$

Chapter 25: Current, Resistance, and Electromotive Force

This chapter transitions from electrostatic situations to moving charges

25.1: Current

Electric current: charges moving from one region to another - circuit: charges moving in closed loop - in electrostatics, conductors were described as having no electric field (and hence no current), but electrons are still free that move randomly throughout the conducting substance (randomness = no net flow) at a fast rate (e.g., 10^6 m/s) - when an electric field exists within the conductor, free electrons move opposite the direction of the field, but are resisted by collisions with the atoms of the conductive material - thus, electrons move ("drift") slowly (e.g., 10^{-4} m/s "drift velocity," v_d) - although the electrons move slowly, the electric field propagates through the conductor body at the speed of light, and all electrons begin moving almost instantaneously - most of the kinetic energy imparted by the field is lost to collisions with the conductor particles that results in heat

Moving particles need not be electrons: can also include positively charged ions, or even "vacancies" or "holes" (areas of electron deficit that act as moving positive charges) - "conventional current" is always defined as the direction of flow of positive charge, and is quantified as the net charge flowing through an area per unit time: $I = \frac{dQ}{dt}$ (even though current has a direction, this is a scalar value) - unit of measurement is an ampere: one coulomb per second

Current flowing through a conductor has two properties: concentration (i.e., the quantity of moving charge in a given volume), and the speed of the individual charges (v_d) - if each of n particles has a charge q, the particles are moving at drift velocity v_d, and the cross-sectional area of the conductor holds A particles, then the total charge moving in a span of time dt is $dQ = n\,q\,v_d\,A\,dt$ - each particle moves a distance $v_d dt$, and current is $\frac{dQ}{dt}$ (measured in amperes), and current density is measured as $J = \frac{I}{A} = n \cdot q \cdot v_d$ (measured as amperes/m^2) - since current is considered to be irrespective of the charges of the particles, the charge is usually represented as $|q|$; alternatively, the vector current density is written as a vector $\overrightarrow{J} = n\,q\,\overrightarrow{v_d}$ (this is because vector current density usually refers to a particular point, while current usually refers to a wire or circuit holistically)

Many types of conductors may have various types of charge flowing - in a closed circuit, the current can remain constant at all cross-sections of the circuit - also, current can be direct or alternating - however, as a general principle, current is composed of n particles, each having q charge, moving with a velocity v_d and a cross-sectional area A, which result in a charge differential $\frac{dQ}{dt}$

25.2: Resistivity

The current density $\overrightarrow{J}$ depends on the electric field $\overrightarrow{E}$ within the conductor, but also on the ease with which charge flows through the conductor; like friction, this property varies by material - Ohm's Law: $\rho = \frac{E}{J}$ - this is an idealized equation that approximates this resistivity - the unit of measurement for resistance is the ohm, $\Omega = \frac{1V}{A}$, and ρ is in ohm-meters: the proportion of the electric field that results in current - an ideal conductor has a resistivity of 1, and an ideal insulator has a resistivity of ∞ - the reciprocal of resistivity is conductivity, which is measured in $\Omega \cdot m)^{-1}$

Material properties: Metals and alloys are good electrical conductors (and also good conductors of heat), while ceramics and plastics are good electrical insulators (and also good insulators of heat) - semiconductors have intermediate resistivity that fluctuates with temperature and impurities, providing control over current - materials may be either ohmic (linear) conductors that provide a constant ρ, or non-ohmic (non-linear) materials with more complex relationships of J and E - however, graphite demonstrates decreasing resistance at higher temperatures (more electrons are freed to move) - superconductivity: some materials exhibit a phase change at very low temperatures that practically eliminates resistivity

Resistivity and temperature: Higher temperatures cause the ions of a conducting material to vibrate with greater amplitude, which further interferes with current and raises resistivity - an approximation of this relationship is $\rho(T) = \rho_0[1 + \alpha(T - T_0)]$, where ρ_0 is resistivity at a reference temperature T_0, and α is the temperature coefficient of resistivity (constant per material) - α is itself affected by temperature differences, but remains approximately constant over modest temperature ranges of 100 C

25.3: Resistance

A conductor with resistivity ρ demonstrates the property $\overrightarrow{E} = \rho\overrightarrow{J}$ - current over a length of conductor flows in the direction of the electric field, and in the direction of decreasing electric potential - the resistance of the conductor is $R = \frac{V}{I} = \frac{\rho L}{A}$, and the canonical version of Ohm's law is $V = IR$ - resistance is measured in ohms (1 volt / ampere) - like resistivity, resistance is affected by temperature: $R(T) = R_0[1 + \alpha(T - T_0)]$ - in ohmic conductors, current and potential are linearly related with a line of slope $\frac{1}{R}$

Diode: A non-ohmic semiconductor that resists current flow in one direction - useful to convert alternating current to direct current

25.4: Electromotive Force and Circuits

A conductor with a steady current must be part of a complete circuit (otherwise, charge will accumulate on one portion of the conductor and cause a resistive electric field) - since most devices cause a voltage drop, some part of the circuit must cause a voltage increase - this is a source of electromotive force (EMF, or ε); not actually a force, but a source of potential energy - standard of potential: the volt ($1\text{V} = \frac{1\text{J}}{1\text{C}}$) - could be a battery, a wall outlet, a solar panel, a generator, etc. - "ideal" source of EMF maintains constant potential difference between terminals that is independent of current

The potential source has an internal electrostatic force $\overrightarrow{F_e}$ (caused by the potential difference between the high-potential terminal and the low-potential terminal), and an internal force $\overrightarrow{F_n}$ that pushes current out of the high-potential terminal - for an ideal source of EMF, $\overrightarrow{F_n}$ is equal and opposite $\overrightarrow{F_e}$, so that the total work done on each charge is zero and the velocity does not change - the work done to move the charge from the low-potential terminal to the high-potential terminal is $W_n = qV_{ab}$, and the potential difference is $\varepsilon = V_{ab}$ - note: current is equal throughout the circuit; does not diminish as potential changes

Non-ideal potential sources exhibit an internal resistance between the terminals - the voltage difference between the terminals thus, $V_{ab} = Epsilon - Ir$ (i.e., the potential drop along the circuit equals the difference in potential of the terminals minus the internal resistance), and $I = \frac{\varepsilon}{R+r}$ - for such potential sources, the EMF equals the difference of potential between the terminals only if no current is flowing

The potential changes around a circuit must be zero: $\varepsilon - I(r + R) = 0$ - for circuits featuring ohmic resistors, the voltage drops and current can be computed algebraically; for non-ohmic resistors, numerical techniques must be used

25.5: Energy and Power in Electric Circuits

In any component on a circuit, the charge passing through it has current I and a voltage drop V_{ab} due to the resistance of the electric field - the current (i.e., speed) of the charge does not change, but the energy carried by the charge is transferred to other forms (e.g., thermal) - however, the current defines the rate at which charge is transferred through the component ($I = \frac{dQ}{dt}$), and the rate at which power is delivered: $P = V_{ab}I$, in the unit of measurement of watts (1 J/s)

For a resistor, $P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R}$ - lost potential is dissipated by the resistor, typically as heat, and transistors typically have a power rating indicating the power that it can dissipate without melting

For an EMF source, the power delivered is $P = V_{ab}I$, and $V_{ab} = \varepsilon - Ir$, so $P = \varepsilon I - I^2r - thisequationindicatesboththegrosspotentialgenerated(\varepsilon I\ )$ and the power lost to internal resistance (I^2r) - however, an EMF source can be connected to another EMF source with stronger EMF (e.g., charging a battery) - power can flow backwards through a battery (from the high-potential connector to the low-potential connector), causing potential energy to be transferred to the EMF source instead of away from it - in this case, $P = V_{ab}I = \varepsilon I + I^2r$

25.6: Theory of Metallic Conduction

In a conductor such as a metal, electrons typically move randomly and experience no net motion, unless an electric field is applied that applies a comparatively small force (in the case of negatively charged electrons, the force is opposite the direction of the field) - resistivity can be written as $\rho = \frac{m}{nq^2\tau}$, where m is the mass of the particle, n is the number of charge carriers per unit volume, and τ is the mean free time that the particle is not encountering a collision

Chapter 26: Direct-Current Circuits

Many types of devices involve circuits that are a complex network of devices - this chapter focuses on the analysis of direct-current (DC) circuits, where the direction of the current does not change - later chapters will analyze alternating-current (AC) circuits

26.1: Resistors in Series and Parallel

Many types of components act in a circuit as a resistor that uses potential for some purpose (e.g., light bulbs and heating elements) - a combination of resistors can be connected in series or parallel to provide different effects - every combination of resistors could be replaced with a single resistor that provides the same total current and potential difference

Resistors in series: As for capacitors in series, the current flowing into each resistor in the series is the same as the the current flowing out of the resistor and into the next resistor - the potential drop of each resistor is $V = IR$ - the resistance of the set of resistors A, B, C is $V = I \cdot (R_A + R_B + R_C)$, and the total resistance is simply the sum: $R_{total} = R_A + R_B + R_C$

Resistors in parallel: Each resistor provides an alternative path for current - the potential difference is the same across each path (because the potential difference between any two points is independent of the path between them) - thus, for a potential drop V_{ab} over a set of resistors A, B, C connected in parallel, the current varies for each resistor: $I_A = \frac{V_{ab}}{R_A}; I_B = \frac{V_{ab}}{R_B}; I_C = \frac{V_{ab}}{R_C}$ - the current is simply the sum of the currents across each resistor: $I = I_1 + I_2 + I_3$, and the total resistance is the reciprocal: $\frac{1}{R_{total}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$ - for the case of two resistors, $R_{total} = \frac{R_1 \cdot R_2}{R_1 + R_2}$ - hence, with resistors in parallel, the total resistance is always less than the resistance of any single resistor in the set - also, at a set voltage, the current varies inversely proportionally to each of them: $V_{ab} = I_1 R_1 = I_2 R_2$, and $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

Comparison with capacitance: Resistors in series add directly because voltage is directly proportional to resistance and current; but capacitors in series add reciprocally because voltage is directly proportional to charge but **inversely** proportional to individual capacitance - similarly, the current for resistors in parallel is inversely proportional to the resistance of each of them, but capacitors add directly because the charge on each is directly proportional to the common voltage

26.2: Kirchhoff's Rules

Many circuits are much more complicated than simple series/parallel combinations - Kirchhoff's rules provide some principles for evaluating them, based on conservation of energy

Kirchhoff's junction rule: The algebraic sum of currents into any junction is zero

Kirchhoff's loop rule: The algebraic sum of potential differences in any loop is zero

Sign conventions: When (conceptually) traveling around a circuit in the direction of conventional current, any gain in potential (i.e., an EMF source) is indicated as a positive EMF ($+\varepsilon$), and any loss in potential (i.e., a resistor) is indicated as a negative EMF ($-IR$) - traveling backwards reverses these signs: traveling backwards through a source produces $-\varepsilon$, and traveling backwards through a resistor produces $+IR$

26.3: Electrical Measuring Instruments

Circuit properties are measured through the positioning of a meter on the circuit - the meter has a spring that experiences torque when exposed to the magnetic field of a current - the degree of deflection of the spring is proportional to the magnetic field, which can be used to measure either voltage or current - each meter has a maximum deflection and a maximum measurement value, but can still be used to measure higher-capacity circuits by placing a resistor in parallel with it, and then factoring the effect of the resistor into the measurements

An ammeter measures current passing through it, altered by a (hopefully minimal) resistance of the spring - the ammeters gets inserted at a point in the circuit, possibly with a resistor in parallel to measure only a portion of the current at that point in the circuit

A voltmeter measures potential difference between two points - placing the voltmeter in parallel with a component causes it to measure the voltage drop of that component - the voltage measured by the voltmeter can be reduced by placing a resistor in series with it

Measuring both voltage and current for the same circuit is difficult, since each one alters charge flow through the circuit and thus affects the readings of the other

An ohmmeter measures resistance of a circuit - a resistor is inserted into the circuit of the ohmmeter, which measures the resistance of the circuit at a fixed voltage, reporting anywhere between infinite resistance (for an open circuit) and 0 (for a shorted circuit)

A potentiometer measures the EMF of a potential source - a potentiometer has a known potential source connected through the full length of a resistor - a second (unknown) voltage source can be connected in the opposite direction through a specified position on the same resistor, and the position is adjusted until the current flow through the second voltage source is zero (i.e., the potential drop of that portion of resistor equals the potential gain across the second voltage source) - hence, the potential contributed by the second voltage source is measurable without actually drawing any current from it

26.4: R-C Circuits

The circuits considered so far reach stability nearly instantaneously - however, capacitors charge over time, depending on the current, which can be altered by resistors (R-C circuit)

Consider a circuit with an ideal power source operating at a constant voltage, a resistor, and a capacitor - at time $t = 0$, current begins flowing: the capacitor has no charge and exhibits no potential difference, so the entire potential difference is determined by the resistor - as capacitor charges, the potential difference across the capacitor increases, and the potential difference across the resistor is also reduced - eventually, the potential difference across the capacitor equals the EMF of the battery source: the potential resistance across the resistor is zero, and no current flows

This circuit exhibits variable potential and current over time - when these variables change, lower-case variable letters are used: $v_{ab} = iR; v_{bc} = \frac{q}{C}$ -using Kirchhoff's loop rule: $\varepsilon - iR - \frac{q}{C} = 0$ - in that formulation, $\frac{q}{RC}$ increases over time, and the final capacitor charge $Q_f = C\varepsilon$

The instantaneous current changes according to a natural logarithm: $i = I_0 e^{-\frac{t}{RC}}$; the instantaneous charge on the capacitor is $q = C\varepsilon(1 - e^{-\frac{t}{RC}})$; and the instantaneous power delivery to the capacitor is $\varepsilon i = i^2 R + \frac{iq}{C}$ (i.e., part of the EMF is dissipated by the resistor and part is stored in the capacitor) - since these values are logarithmic, the charge never actually reaches zero, but gets asymptotically close - the quantity $\tau = RC$ defines the time constant, or relaxation time (in seconds), of the circuit: small τ = fast-charging capacitor; large τ = slow-charging capacitor

Removing the potential source from the R-C circuit (but keeping a closed loop including the resistor) causes current to flow again, but to flow backwards (from the positive plate to the negative plate), and eventually reach zero: $i = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$, and the instantaneous charge in the capacitor is $q = Q_0 e^{-\frac{t}{RC}}$

The total energy stored in the battery is εQ_f - this is also the amount of energy dissipated by the resistor, irrespective of C, R, or ε

26.5: Power Distribution Systems

In typical consumer electronics, automobiles often use DC, while households use AC - devices are wired in parallel to prevent an outage or disconnection in one device from affecting other devices - current runs from the power source to a "hot" wire that is split in parallel to each device, and then from each device into a neutral wire that connects back to the power source through a ground electrode

In the U.S., the potential supplied by each outlet is 120 volts; each device provides a resistance, thus inducing a specific current, and consuming power at a power input $P = VI$ - simpler devices exhibit a lower resistivity when cold (i.e., first turned on) than later (e.g., light bulbs start out dim and warm up), and at first experience a surge of current that diminishes as device warms up (e.g., why light bulbs burn out when first turned on from cold)

Important Formulae

- Current: $I = n\left|q\right|v_d A$
- Current Density: $J = nqv_d$
- Resistivity: $\rho = \frac{E}{J}; R = \frac{\rho L}{A}$
- Current: $I = \frac{V}{R} = JA$
- Power: $P = V_{ab}I$
- R-C Circuit Charging: $q = Q_f(1 - e^{-\frac{t}{RC}})$
- R-C Circuit Instantaneous Current: $i = I_0 e^{-\frac{t}{RC}}$
- R-C Circuit Discharging: $q = Q_0 e^{-\frac{t}{RC}}$

Chapter 27: Direct-Current Circuits

Magnetic fields operate like electric fields, but only operate on electric charges in motion - this chapter covers magnetic interaction; chapter 28 covers magnetic field creation - the relationship between electric and magnetic fields is later reviewed as Maxwell's equations

27.1: Magnetism

Some substances exert a permanent magnetic field, and interact with other magnetic fields even at rest - these substances exhibit a polarity such that the "magnetic north pole" end is attracted to the "magnetic south pole" end of another magnet, and will even reorient at rest - other substances (particularly metals) can exhibit an induced magnetic polarity

The Earth acts as a permanent magnet, with the magnetic south near the north pole - the center is not directly aligned with geographic north (the discrepancy is called the magnetic declination), and is also not aligned perfectly with the horizon (the discrepancy is called magnetic inclination)

Magnetic poles always exist in pairs; a magnetic monopole has never been discovered in nature (e.g., if a magnetized bar is broken in half, the ends at the break become a new magnetic north and magnetic south) - however, magnetic and electric forces are related: electric current creates a magnetic field, and moving a magnet near a conducting loop causes a current in the loop - magnetic forces are due to the interactions of moving electrons in the atoms of a magnetic body - permanent magnets exhibit a coordinated motion of particular atomic electrons, while these interactions are uncoordinated in unmagnetized bodies

27.2: Magnetic Field

Magnetic interactions can be described in two steps: (1) a moving charge or current creates a magnetic field in surrounding space (in addition to electric field), and (2) the magnetic field exerts a force $\overrightarrow{F}$ on any other moving charge or current present in the field

A magnetic force ($\overrightarrow{B}$) on a charged, moving particle, traveling in the direction $\overrightarrow{v}$, has four characteristics:

- Magnitude of force is directly proportional to magnitude of charge of particle
- Magnitude of force is directly proportional to magnitude of magnetic field
- Magnitude of force depends on magnitude of $\overrightarrow{v}$
- Direction of force is perpendicular to both $\overrightarrow{B}$ and $\overrightarrow{v}$ - "right-hand rule": placing the vectors $\overrightarrow{B}, \overrightarrow{v}$ tail to tail, and rotate $\overrightarrow{v}$ toward $\overrightarrow{B}$; curling the fingers of the right hand in this direction, the thumb points in the direction of the magnetic force on the particle (note: this is for positively charged particles; negatively charged particles experience a force in the opposite direction)

All four characteristics are represented as: $\overrightarrow{F} = q\overrightarrow{v} \times \overrightarrow{B}$ (note: when q is negative, the force is in the opposite direction), and $F = |q|vB\perp$ - the strength of the magnetic field is measured in tesla (1 T = 1 N/A $\cdot$ m) and the gauss (1 G = 10^{-4} T); the magnetic field of the earth is about 1 gauss

A magnetic field can be detected and measured according to the deflection of a moving test charge - no deflection occurs if the charge is moving parallel to the magnetic field

Where both magnetic and electric fields are present in the same space, a charged particle experiences both as a vector sum: $\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}$

27.3: Magnetic Field Lines and Magnetic Flux

A magnetic field can be illustrated as magnetic field lines, illustrating the direction of force exerted on a magnetic north pole - however, unlike electric fields, the magnetic field line does not point in the direction of magnetic force, but in the direction of the magnetic field: the direction of the magnetic force is always perpendicular to the magnetic field line

Magnetic flux (Φ_B): like electric flux, magnetic flux represents the component of a magnetic field normal to the surface - for each unit of the area dA of the surface, the flux is $B_\perp = B\cos(\phi)$ - the magnetic flux through this area is $d\Phi_B = B_\perp\,dA = B\cos(\phi)\,dA = \overrightarrow{B}\cdot d\overrightarrow{A}$, and the total flux over the surface is $\Phi_B = \int B_\perp\,dA = \int \overrightarrow{B}\cdot d\overrightarrow{A}$ - this is a scalar quantity, in units of the weber: 1 Wb = 1 T $\cdot$m^2 = 1N $\cdot$ m/A - because magnetic monopoles do not exist, the total magnetic flux through a closed surface is always defined as zero (Gauss's law for magnetism) - also, magnetic field lines do not end, but always continue through the body of the magnet to form a closed loop

27.4: Motion of Charged Particles in a Magnetic Field

A charged particle moving perpendicularly in a magnetic field is subjected to a magnetic force that is perpendicular to its direction of travel - because it is perpendicular, the magnetic field cannot change the magnitude of its direction, but only its direction - accordingly, the magnetic field can never do work on the particle - if moving through a magnetic field at a constant velocity, the particle experiences a centripetal force, and thus travels in a circle - in this case, $\frac{F=mv^2}{R}$, and $R = \frac{mv}{|q|B}$ - the angular speed of the particle is $\omega = \frac{v}{R} = \frac{|q|B}{m}$, and the particle makes $f = \frac{\omega}{2\pi}$ revolutions per unit time (this is the cyclotron frequency) - if the particle does not travel perpendicularly through field but at an angle, then the particle moves in a helix

A nonuniform magnetic field can produce various types of motion - magnetic bottle: two circular coils separated by a distance; particles near either coil spiral in toward the center, and oscillate on this path between the two coils - can be used to confine a set of particles, or even a plasma with a temperature of 10^6 K

27.5: Applications of Motion of Charged Particles

Velocity selector: A beam of charged particles may be formed where the particles are traveling at different velocities - electric and magnetic fields can be used to form a velocity selector to filter out all particles not traveling at a desired speed - two electrified plates are placed near the beam and charged, and a magnetic field is initiated through the plates - the magnetic field pushes particles in the opposite direction of the electric field, proportional with the velocity of the particle - hence, only particles with a desired speed $v = \frac{E}{B}$ pass through - this principle was used to measure the mass of the electron

Mass spectrometer: A device using a similar crossed-field configuration - when a beam of particles passes through the electric plates, the magnetic field induces an arced path proportional to the mass of the particles, which is detected by a screen that fluoresces at the point of contact; very precise way of measuring mass

27.6: Magnetic Force on a Current-Carrying Conductor

Motors work by exerting a magnetic force on a moving current - the force on each charged particle is $\overrightarrow{F} = q\overrightarrow{v_d} \times \overrightarrow{B}$, and the total force is $\overrightarrow{F} = I\,\overrightarrow{l} \times \overrightarrow{B}$ - for a non-straight wire, the force can be computed in segments: $d\overrightarrow{F} = l\,d\,\overrightarrow{l} \times \overrightarrow{B}$ - direction does not change even if charge of particle changes: for negative charges, both q and the direction of force change and cancel out

In a loudspeaker, a coil carries an audio signal as an oscillating current - permanent magnets positioned inside and outside the coil create an outward-radiating magnetic field - the interaction of the oscillating current and the static magnetic field causes an oscillating force on the cone of the speaker that also oscillates and produces louder sound - amplifying the current causes an equivalent amplification of the magnitude of oscillation of the coil, the force, the oscillation of the cone, and the production of sound

27.7: Force and Torque on a Current Loop

Consider a conducting rectangle with current running counterclockwise, oriented in a magnetic field - when the plane of the rectangle is normal to the magnetic field, the magnetic field force on each side of the rectangle is equally balanced by the magnetic field force on the other side in the other direction, so no net force results - however, when the rectangle is rotated from normal (such that the magnetic field passes through the plane of the rectangle at an angle, or, maximally, when the plane of the rectangle is parallel to the magnetic field), the sides experience a magnetic force with opposing direction and magnitude, but at an angle that produces a torque with magnitude $\tau - I\dot{B}\, A\, \sin(\phi)$

In these situations, the quantity $\mu = IA$ represents the dipole moment, and simplifies to $\tau = \mu\, B\, \sin(\phi)$, and $\overrightarrow{\tau} = \overrightarrow{\mu} \times \overrightarrow{B}$

When a dipole experiences a force and changes orientation, the field is doing work on the dipole - $dW = \tau d\phi$ - for an electric field, the torque $\overrightarrow{\tau} = \overrightarrow{p} \times \overrightarrow{E}$, and the potential energy $U = -\overrightarrow{p} \cdot \overrightarrow{E}$; the same is true for magnetic fields: torque $\overrightarrow{\tau} = -\overrightarrow{\mu} \times \overrightarrow{B}$, and the potential energy $U = -\overrightarrow{\mu} \cdot \overrightarrow{B}$

Solenoid: A set of (N) coils positioned perpendicular to a magnetic field - running current through the loop causes a torque that tends to rotate it to cause the axis through the solenoid to be parallel to the magnetic field, with torque $\tau = N\, A\, I\, B\, \sin(\phi)$

Galvanometer: A metering device where a current running through a conducting wire causes torque on a needle that rotates to show the magnitude of the current

Magnetic resonance imaging: A patient is positioned in a magnetic field that orients dipolar hydrogen atoms within tissue - a radio wave disrupts this orientation, and the absorption of the radio wave can be measured as a degree of absorption, which reflects the hydrogen (i.e., water) content of the tissue

Magnetic dipoles: A permanent magnet is a material with atoms exhibiting a permanent orientation of dipole moment - a permanent magnet can temporarily magnetize other materials having a flexible dipole moment: the magnetic field causes its atoms to reorient, and the resulting dipole is magnetically attracted to the permanent magnet - adding kinetic energy (e.g., by tapping or heating the temporarily magnetized material) causes a reorientation of dipole moment and a loss of magnetic properties

27.8: The Direct-Current Motor

DC electric motor: An open-loop conductor is positioned to rotate between two permanent magnets, and is in contact with a second open loop including an EMF - as the EMF pushes current through the rotor, the magnetic field exerts a torque on the rotor that turns it - when the rotor is parallel with the magnets, maximum torque is exerted - if the rotor remained in closed contact with the second loop, the rotor would rotate to a perpendicular state between the magnets, and remain there in equilibrium (since rotating in either direction pushes it back to perpendicular - however, at this point, commutators at the ends of the rotor cause it to contact both ends of the second open loop, causing no potential to flow through the rotor, and the current and magnetic force cease - inertia carries the rotation of the rotor through this point, and the commutators are now connected to the opposite terminals of the EMF, causing current to flow in the other direction around the rotor (not exactly alternating, because the current always goes in one direction, e.g. counterclockwise, around the rotor)

The torque and power of the motor can be increased by using many rotors, and/or by applying a stronger magnetic force than a permanent magnet can exert (e.g., using an electromagnet instead)

Electric motor power: As usual, $P = V_{ab}\,I$ - the resistance that creates potential results primarily from the force of the magnetic field on the current ("back" EMF or "induced" EMF): $V_{ab} = \varepsilon + I\,r$ (where ε is the back EMF, and r is the internal resistance of the rotor)

27.9: The Hall Effect

When a current runs lengthwise through a conducting strip and a magnetic field passes through the strip, charge gathers on the top of the strip - if the charge carrier is positive, the charge on top of the strip is negative, creating an electric field within the strip - the charge builds up until the electric field counters the magnetic force - this can occur even if the charge carrier is positive: the charge that builds up on the strip is not a positively charged ion, but a "hole" representing the absence of an electron - the strip therefore picks up a negative charge along one edge (excess electrons) and a positive charge along the other edge (holes); Hall voltage: the potential difference between the edges - Hall effect: $n\,q\,E_z = -J_x B_y$ - this can be used to compute the quantity nq, or, if the charge density is known, v_d

Chapter 28: Sources of Magnetic Field

Magnetic fields can be created by both permanent magnets and electric currents in electromagnets

28.1: Magnetic Field of a Moving Charge

A moving point with charge q and constant velocity $\overrightarrow{v}$ creates a magnetic field - the field created by this source point can be measured at a field point - similarly to electric fields, the magnetic field is proportional to the strengths of the charges and the distance between the particles, but is perpendicular to the plane containing the line and the velocity vector of the source point: $B = \frac{\mu_0}{4\pi}\frac{|q|\,v\,\sin(\phi)}{r^2}$, where $\frac{\mu_0}{4\pi}$ is a proportionality constant - the magnetic field vector is $\overrightarrow{B} = \frac{\mu_0}{4\pi}\frac{q\,\overrightarrow{v}\times\hat{r}}{r^2}$, where $\hat{r}$ is a unit vector from the source point to the field point - along the vector in the direction of travel of the field point, the magnetic field is zero

The proportionality constant $\mu_0 = 4\pi \times 10^{-7} T \cdot meters/amperes$ - this value is related to ϵ_0 by the speed of light: $c^2 = \frac{1}{\epsilon_0 \mu_0}$

Note: These equations are for charges moving with constant velocity - when charge accelerates, the magnetic field is different - this occurs, e.g., in the bend of a conducting wire, but the difference is proportional to $(v_d)^2$ which is very small, so it is disregarded here

28.2: Magnetic Field of a Current Element

As with electric fields, the magnetic field at a field point is the sum of the individual magnetic fields caused by each moving charge - Biot-Savert Law: for a current moving through a sequence of segments of conducting wire, the magnetic field created is $\overrightarrow{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\, \overrightarrow{l} \times \hat{r}}{r^2}$ - it's not possible to experiment with just a single segment, but the total magnetic field of several segments can be examined - if other materials are present near the conductor, the magnetization of these materials; however, this value is negligible (and disregarded here) for materials that aren't ferromagnetic materials (e.g., iron) or superconducting

28.3: Magnetic Field of a Straight Current-Carrying Conductor

Exemplary application of Biot-Savart to a straight conductor: Imagine a conducting wire positioned along the y-axis from $-a$ to a, with current running vertically upward - for a field point $(x, 0)$, the magnetic field produced by every point will have a direction through the field point and into the z plane; thus, the magnitude of the total magnetic field is the sum of the magnetic field created by every point - thus, $B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x\, dy}{(x^2+y^2)^{\frac{3}{2}}}$ - for a comparatively long wire, this becomes $B = \frac{\mu_0 I}{2\pi x}$ - this is true for every point around the cylinder surrounding the conducting wire, and for each circle of radius r, the magnitude is the same (though the direction changes)

28.4: Force Between Parallel Conductors

When two current-carrying conductors are in proximity, each conductor exerts a magnetic field that affects the other - if two wires run in parallel, one above the other, the bottom wire creates a magnetic field oriented laterally perpendicular to the top wire, and the magnetic field exerts a downward magnetic force on the current of the top wire - similarly, the top wire produces a magnetic field oriented laterally perpendicular to the bottom wire, and the magnetic field exerts an upward magnetic force on the current of the bottom wire - thus, when oriented parallel with current running in the same direction, the wires create a magnetic field that pulls them together - if the direction of current in one wire is reversed, the magnetic field of each wire exerts a repulsive magnetic force on the other wire

The ampere is defined in terms of the magnetic force: if present in each of two parallel conductors of infinite length and one meter apart, and if each wire carries a current of one ampere, each wires induces a magnetic force of 2×10^{-7} newtons per meter on the other

28.5: Magnetic Field of a Circular Current Loop

Consider current being conducted around a closed circular loop of radius a, centered at the origin in the (y, z) plane - the magnetic field at a point $(x, 0)$ is $dB = \frac{\mu_0 I}{4\pi} \frac{dl}{x^2+a^2}$ - the (y, z) components of the magnetic field created by each point are exactly canceled by the (y, z) component of the magnetic field created by the point directly across from it on the loop (since current is moving in the opposite direction) - however, the x component from each point on the loop adds - the sum is the axis through the center of the coil, with the direction defined by the right-hand rule (coiling the fingers in the direction of current travel around the coil, the thumb points in the direction of the magnetic field), and the magnitude of the field is $B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{\frac{3}{2}}}$ - if n coils are used, then along the center axis of the loop, the magnitude of the magnetic field is $B = \frac{\mu_0 n I}{2a}$

Chapter 27 presented the concept of the magnetic moment of a current-carrying loop ($\mu = IA$) indicating the sensitivity of the loop to a magnetic current - however, the magnetic field created by the current-carrying loop is also directly proportional to its magnetic moment: $B = \frac{\mu_0 \, \mu}{2\pi(x^2+a^2+^{3over2})}$

28.6: Ampere's Law

Just as Gauss's law simplifies determinations of the electrical field for a surface, Ampere's law simplifies determinations of the magnetic field - this law states that along a path, the net magnetic field is $\oint \overrightarrow{B} \cdot d\overrightarrow{l}$

For a long, straight conductor, the magnetic field at any point on a circle of radius r is $B = \frac{\mu_0 I}{2\pi r}$ - for the path around the entire circle, the magnitude is $\oint \overrightarrow{B} \cdot d\overrightarrow{l} = B \oint dl = \mu_0 I over 2\pi r(2\pi r) = \mu_0 I$ - reversing the current results in $B = -\mu_0 I$ - these line integrals also follow the right-hand rule: if curling the fingers around the circle in the direction of integration, the thumb points in the direction of a positive magnetic field

Ampere's Law: $\oint \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 I_{\text{encl}}$ - i.e., the net magnetic field over a path only depends on the net magnitude of current moving within the path - currents outside of the path, and the shape and size of the path, do not matter - as with Gauss's Law, the magnetic field on each point of the path enclosing no current may be nonzero, but the sum around the path must be zero - this formulation of the law only applies if the current does not vary and if no magnetic materials are present; generalizations that include other conditions follow - also, Ampere's Law does not state that the magnetic force is conserved around the path: it isn't, since it depends on velocity

28.7: Applications of Ampere's Law

(examples presented)

28.8: Magnetic Materials

Magnetic fields are created by the circulation of electrons within the body of a material that create current loops - typically, such loops are randomly aligned, resulting in no net field; but in some materials, an external magnetic field can reorient the loops in a synchronized direction, thus causing the material to become magnetized - moreover, the period and shape of these current loops are quantized according to a value called Bohr's magneton: $\mu_B = 9.274 \times 10^{-24} A \cdot m^T 2$ - can also be influenced by electron spin

Paramagnetism: Atoms of paramagnetic materials have a net magnetic moment of μ_B, and when placed in a magnetic field, these atoms experience a torque of $\overrightarrow{\tau} = \overrightarrow{\mu} \times \overrightarrow{B}$ - this torque aligns the magnetic moment of the atoms with the field, and the current loop of the atom then adds to the magnetic field - the ratio of the additional $\overrightarrow{B}$ field with the original field is the magnetization ($\overrightarrow{M}$) of the material: $\overrightarrow{M} = \frac{\mu_{total}}{V}, and the additional strength of the magnetic field is \overrightarrow{B} = \overrightarrow{B}_0 + \mu_0 \overrightarrow{M}$ - note: the unit of $\mu_0 \overrightarrow{M}$ is the tesla - within any paramagnetic material, the magnetic field is increased by a factor known as the relative permeability of the material (K_m), which typically ranges from 1.00001 to 1.003, and the magnetic field is then computed according to $\mu = K_m \mu_0$ -

Curie's Law: Paramagnetism is inversely proportional to temperature (which disrupts the reorientation of the electron orbits): $M = \frac{CB}{T}$, where M is the magnetization of the material, B is the strength of the field, T is the temperature, and C is a material-specific constant (Curie constant)

Diamagnetism: Diamagnetic materials behave opposite to paramagnetic materials: the presence of a magnetic field causes electron loops to reorient in a manner that opposes the magnetic field - the relative permeability K_m of these materials is typically within the range of 0.9999 to 0.99999 - unlike paramagnetic materials, diamagnetism is not typically proportional to temperature

Ferromagnetism: In some materials, the atomic structure causes a localized synchronization of electron loops in a "magnetic domain," even in the absence of a magnetic field - while the magnetic domains themselves are randomly oriented, the sizes and shapes of the domains are affected by the presence of a magnetic field: domains that are aligned with the magnetic field grow - these effects are proportional to the strength of the magnetic field, all the way up to a saturation magnetization - the resulting permeability K_m is on the order of 1,000 to 100,000 - a magnetic field attracts ferromagnetic materials (e.g., iron) much more strongly than paramagnetic materials (e.g., aluminum)

Hysteresis: Many ferromagnetic materials will permanently retain this alignment even when the magnetic field dissipates (i.e., the electrons are oriented into hysteresis loops) - this effect can be reversed only by applying the opposite magnetic field, which dissipates energy and releases heat - these properties vary by material, and it is often desirable to find materials that exhibit hysteresis only over a narrow range (e.g., small positive magnetic field induces hysteresis, and small negative magnetic field removes hysteresis), as this is faster and more energy-efficient for applications such as computer memory

Chapter 30: Inductance

When a coil is positioned in a circuit, the current running through the coil creates a magnetic field through the center of the coil - due to inductance, this magnetic field can affect other circuits and vice versa (mutual inductance), as well as other components of the same circuit (self-inductance)

30.1: Mutual Inductance

Inductance only arises when a magnetic field changes - this can arise from changes in current flowing through a circuit

Consider two conducting coils positioned near (parallel to) each other, respectively carrying current i_1, i_2 - when i_1 changes, the magnetic field created by i_1 changes and induces an EMF in the second coil: $\varepsilon_2 = -N_2 \frac{d\Phi_{B2}}{dt}$, where N_2 is the number of loops in the second coil - this can be written as $N_2 \Phi_{B2} = M_{21} i_1$, where M_{21} is a proportionality constant representing the relationship between the magnitude of the changing current $\frac{di_1}{dt}$ and the induced EMF in the second loop in terms of $\frac{di_1}{dt}$ - $N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$, and $\varepsilon_2 = -M_{21} \frac{1}{dt}$ - as it happens, this constant also represents the inverse relationship, i.e., the effect of changing current through the second coil on the induced EMF in the first coil: $\varepsilon_2 = -M \frac{di_1}{dt}$ and $\varepsilon_1 = -M \frac{di_2}{dt}$ - thus, $M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$ - the induction follows Lenz's law, i.e., the direction of the induced current creates a magnetic field that opposes the inducing field

The SI unit of mutual inductance is the henry: $1H = \frac{1Wb}{1A} = \frac{1V \cdot s}{1A} = 1Ohm \cdot s = \frac{1J}{A^2}$

Mutual inductance can disrupt the expected operation of a circuit, and be reduced by insulating coils from each other, positioning at a distance, or inserting insulation - alternatively, mutual inductance can be used advantageously: in a transformer, a variable alternating current in a first coil induces an alternating current in a second coil with a potentially different current, based on M

30.2: Self-Inductance and Inductors

In addition to interaction between coils in different circuits, induction in a coil in a circuit can affect the same circuit - the self-inductance of the circuit operates similarly to mutual inductance, but using the constant L instead of M: $L = \frac{N\Phi_B}{i}$, and $N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$ - the self-induced EMF is $\varepsilon = -L\frac{di}{dt}$ - the sign indicates that the EMF opposes the change in the current - note that Kirchhoff's loop law still holds: for a circuit at any point in time, the potential gains and drops equal zero, including the EMF induced by inductors

An inductive coil positioned in a circuit is called an inductor or a choke, with various effects: the inductor induces a negative EMF while current is increasing; the inductor induces a positive EMF while current is decreasing; and the inductor reduce the amplitude of alternating current

Uses of inductors: Fluorescent bulbs are created by ionizing a gas (producing a plasma) - however, ionization causes the plasma to become a non-ohmic conductor with decreasing resistance as current increases - as a result, a high current could cause an escalation of current through the plasma that damages the bulb - instead, an inductor is included as a magnetic ballast that resists large changes in current - additionally, the ballast allows ionization to be maintained by alternating current; otherwise, the ionization of the plasma quickly ends when the current drops to zero during alternation

Self-inductance depends on the size, shape, number of turns, and magnetic properties of nearby materials: a ferromagnetic core, such as iron, can increase self-inductance by a factor of 5,000 - however, inductance is also affected by the saturation of the material, resulting in non-linear inductance that is not dealt with here

30.3: Magnetic-Field Energy

While the current of a circuit is increasing, the potential consumed by the inductor is used to produce the magnetic field of the inductor - the magnetic field thus stores the energy consumed while creating the magnetic field, and releases the energy back into the circuit (as the EMF) when the current drops - the rate at which power is being stored in the magnetic field of the inductor is $P = V_{ab}\, i = L\, i\, \frac{di}{dt}$, and the total energy stored in the magnetic field is $U = \frac{L\, I^2}{2}$

The magnetic field is much stronger within the core of the inductor than anywhere outside - the energy density of the magnetic field is $u = \frac{B^2}{2\mu}$, where μ is the magnetic permeability of the material in the core, or μ_0 if a vacuum is used

30.4: The R-L Circuit

Charging an inductor: In a circuit combining a resistor and an inductor, the current is initially zero - when the circuit is connected to an EMF source and closed, the current that would typically reach a stable value instantaneously instead varies over time - the voltages around the circuit sum to zero, so the voltage across the resistor is $v = iR$, and the voltage across the inductor is $v_{bc} = L\frac{di}{dt}$, and the total loop is $\varepsilon - iR - L\frac{di}{dt} = 0$ - also, $\frac{di}{dt} = \frac{\varepsilon - iR}{L}$ - the final current does not depend on the inductor, but is simply $I = \frac{\varepsilon}{r}$ - the instantaneous current at any time is $i = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$ - similar to capacitors, the time constant $\tau = \frac{L}{R}$ indicates the rate at which the circuit reaches its final current (specifically, the number of seconds at which the circuit reaches 63% of its final value) - the instantaneous rate of power delivery is $\varepsilon i = i^2 R + Li\frac{di}{dt}$

Discharging an inductor: When the potential source is disconnected from the circuit (but the circuit remains closed), the inductor discharges in a mirror process to charging the inductor: $i^2 R + Li\frac{di}{dt} = 0$, and $i = I_0 e^{-\frac{R}{L}t}$

30.5: The L-C Circuit

A circuit combining an inductor and a capacitor demonstrates oscillation of current and charge - when the capacitor is fully charged with an EMF source that is then disconnected, the capacitor first discharges into the circuit (from the positive plate to the negative plate) through the inductor, which creates a magnetic field - in the absence of the inductor, the capacitor would instantly reach a high current that slowly diminishes; but because the inductor resists fast current changes, the opposite happens: the current is initially small and steadily rises until the moment that the capacitor is fully discharged - then, the inductor begins discharging into the circuit in the same direction, thus recharging the capacitor - however, the capacitor recharges in the opposite direction with the plates reversed - when the inductor is depleted, the capacitor begins discharging into the circuit again, but creating a current in the opposite direction, which recharges the inductor - again, when the capacitor is depleted, the inductor discharges into the circuit again, thus recharging the capacitor, and the process repeats

In L-C circuits, the charge oscillates between the first plate, the magnetic field of the inductor, the second plate, the magnetic field of the inductor, and back to the first plate - this circuit demonstrates a harmonic oscillation with an angular frequency $\omega = \sqrt{\frac{1}{LC}}$, and a linear frequency $f = \frac{\omega}{2\pi}$ - the current at any time is $i = -\omega Q \sin(\omega t + \phi)$, where Q is the initial charge of the capacitor - also, the instantaneous charge on the capacitor is $q = Q\cos(\omega t + \phi)$, and $i = \pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q^2}$

30.6: The L-R-C Series Circuit

The L-C circuit summary above disregarded resistance - taking resistance into account, the oscillation is dampened - a small resistance causes underdamping (slowly decaying oscillation), where $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ - an increasing resistance causes a faster rate of decay - at $R^2 = \frac{4L}{C}$, the system does not oscillate, and the circuit is critical damping - at an even larger resistance causes overdamping (no oscillation and slow rate of discharge from capacitor)

Chapter 32: Elecromagnetic Waves

An electromagnetic wave is a self-sustaining combination of electric and magnetic fields, propagating as a combination of sinusoidal functions, and carrying both energy and momentum - various types of electromagnetic waves can exist, differing in amplitude and frequency

32.1: Maxwell's Equations and Electromagnetic Waves

According to Faraday's Law, a time-varying magnetic field creates an electric field; and according to Ampere's Law, a time-varying electric field creates a magnetic field - thus, when either an electric or magnetic field changes with time, a field of the other type is created in adjacent regions of space, enabling the propagation of the change

Early study of electromagnetic theory led to the discrete study of electrostatics and magnetic fields created by current, with a relationship that precisely equaled the speed of light - Maxwell first recognized the interrelationships of these fields through Maxwell's equations

As a result of Maxwell's equations, every acceleration of charge produces electromagnetic waves - harmonic oscillation of a point charge causes continuous acceleration, and therefore continuous emission of electromagnetic waves radiating away from the point charge; the emissions are strongest 90 degrees from the axis, and are zero along the axis

The unification of electromagnetic theory was verified by Heinrich Hertz, who determined that $v = \lambda f$, and that the speed of electromagnetic waves equaled the speed of light

The wavelengths with which electromagnetic waves propagate, and the properties exhibited at various ranges of wavelengths, define the electromagnetic spectrum - a small frequency range (380 to 750 nm) comprises visible light - the specific wavelength is perceived as color; white light includes all visible wavelengths, and specific wavelengths can be filtered from visible light to create monochromatic light - lower frequencies (long wavelengths) are used for radio and television broadcasts, while higher frequencies (short wavelengths) are used for high-energy applications, such as X-ray imaging

32.2: Plane Electromagnetic Waves and the Speed of Light

Plane wave: Consider an X-axis of travel in space, and a plane moving perpendicular to the axis - at every point behind the plane, a uniform electric field is created along one axis (e.g., oriented upward when viewing the plane from behind), and a uniform magnetic field is created along the other axis (e.g., oriented rightward when viewing the plane from behind) - the created fields are thus transverse to the direction of travel

A plane wave complies with all of Maxwell's equations:

- Gauss's laws: Any surface encloses no electric charge or magnetic element, and the magnetic and electrical flux are both zero
- Faraday's Law: Any shape oriented perpendicular to the magnetic field will exhibit a positive magnetic flux therethrough equaling the opposite of the sum of the electric field around its perimeter, and the strength of the electric field is $E = c\,B$)
- Ampere's Law (any shape oriented perpendicular to the electric field will exhibit a positive electric flux therethrough equaling the opposite of the sum of the magnetic field around its perimeter)

Faraday's Law and Ampere's Law only agree if electromagnetic waves travel at speed $c = \frac{1}{\sqrt{e_0 \mu_0}}$, i.e., the speed of light

Key properties of electromagnetic waves:

- Electromagnetic waves are transverse: $\overrightarrow{E}$ and $\overrightarrow{B}$ are both perpendicular to the direction of wave propagation, as well as each other ($\overrightarrow{E} \times \overrightarrow{B}$)
- The magnitudes of $\overrightarrow{E}$ and $\overrightarrow{B}$ are related: $E = cB$
- Electromagnetic waves travel through a vacuum with a definite and unchanging speed, and need no medium

More accurate description of electromagnetic waves: Consider a series of planes moving along the x-axis - between each pair of planes, the electric and magnetic fields are the same at all points, but the fields differ for different pairs of planes - if the sequence of variations is sinusoidal, and if the planes are taken to a limit such that the variations are continuous, the model begins to resemble actual electromagnetic waves

Polarization: Electromagnetic waves traveling in a particular direction always exhibit $\overrightarrow{E}$ and $\overrightarrow{B}$ fields oriented perpendicularly (specifically, the magnetic field is always oriented 90 degree clockwise with respect to the electric field) - however, the actual orientation of the fields with respect to the direction of travel gives rise to the polarity of the wave - a wave is linearly polarized if the electric field is always oriented along a particular axis

32.3: Sinusoidal Electromagnetic Waves

In a sinusoidal electromagnetic wave, the fields $\overrightarrow{E}$ and $\overrightarrow{B}$ vary sinusoidally, and the spatial variation of the fields is also sinusoidal - the wavelength and frequency are related by $c = \lambda\, f$ - specifically, as the electromagnetic wave travels along the axis of propagation, the electric field along the axis behind it varies as $\overrightarrow{E}(x,t) = \hat{j}\, E_{max} \cos(kx - \omega t)$, and the magnetic field varies as $\overrightarrow{B}(x,t) = \hat{k}\, B_{max} \cos(kx - \omega t)$ - for a plane wave (i.e., a planar electromagnetic wave traveling together), these fields do not just exist at one line parallel to the direction of propagation, but are uniform at every trailing plane parallel to the planar wave - these sinusoids are in phase, and $E_{max} = c\, B_{max}$

Electromagnetic waves in matter: When traveling through a medium, the speed of light becomes $v = \frac{1}{\sqrt{\epsilon\, \mu}}(where \epsilon$ is the permittivity of the medium and μ is the electromagnetic permeability), or $v = \frac{c}{\sqrt{K\, K_m}}$ (where K is the dielectric constant and K_m is the relative permeability of the medium) - for any medium, the index of refraction $n = \frac{c}{v} = \sqrt{K\, K_m}$ - however, the actual relative permeability of the medium is not experienced by the electromagnetic wave, because it is traveling much faster than the molecules can reorient; rather, the wave experiences only a small fraction of K_m

32.4: Energy and Momentum in Electromagnetic Waves

The energy density of an electric field at a point in space is $u = \frac{1}{2}\epsilon_0 E^2$, and the energy density of a magnetic field at a point in space is $u = \frac{1}{2\mu_0}B^2$ - thus, at a point where electric and magnetic field exist due to an electromagnetic wave, $u = \frac{1}{2}(\epsilon_0 e^2 + \frac{B^2}{\mu_0})$, or $u = \epsilon_0 E^2$ - i.e., the energy density of the magnetic field and the electric field are identical

Electromagnetic energy flow: Considering a stationary plane oriented parallel to the plane wave, the energy that is transferred through the plane can be measured in (energy / (unit area * unit of time)), or watts / $m^2 - the magnitude is S = \epsilon_0 \, c \, E^2 = \frac{E \, B}{\mu_0}$ - a vector describing both the magnitude and the vector, the Poynting vector, is defined as $\overrightarrow{S} = \frac{1}{\mu_0} \overrightarrow{E} \times \overrightarrow{B}$, which of course is always oriented in the direction of wave propagation - the total energy flow per unit time through any closed surface is $P = \oint \overrightarrow{S} \cdot d\overrightarrow{A}$ - since the magnitude of the Poynting vector varies with respect to the wave frequency, it's helpful to consider the average value, or intensity: $I = S_{av} = \frac{(E_{max})^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}(E_{max})^2 = \frac{\epsilon_0 c}{2}(E_{max})^2$ - if traveling through a medium, simply replace ϵ_0, μ_0 with ϵ, μ

Electromagnetic waves also transport momentum, with a momentum density (measured in momentum per unit volume): $\frac{dp}{dV} = \frac{S}{c^2}$ - the flow rate of momentum through an area A is $\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$ - substituting $S_{av} = I$ for S yields an average flow rate - if the electromagnetic wave is absorbed, the radiation pressure exerted on the surface is $p_{rad} = \frac{S_{av}}{c} = \frac{I}{c}$; but if the electromagnetic wave is reflected, the radiation pressure $p_{rad} = \frac{2I}{c}$ - while the magnitude of radiation pressure is typically very small, it becomes significant in places like the interior of stars

32.5: Standing Electromagnetic Waves

An electromagnetic wave can be reflected to produce a standing wave - for example, a wave may encounter a planar surface that is a perfect conductor - because it is a conductor that does not contain a charge, the electric field parallel to the plane must be zero - this is achieved because the electromagnetic field induces an oscillating current within the conductor that produces an opposing electric field, leading to momentary local electric fields that sum to zero - the induced currents also create a magnetic field that complements the magnetic field - the conductor also reflects the electromagnetic wave, producing a standing wave - at the conducting surface, the electric field is zero, and thus a node of the standing wave; but the magnetic field is maximum, and thus an antinode of the standing wave - this aspect occurs throughout the standing wave: at $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$, the electric field exhibits a nodal plane, but the magnetic field exhibits an antinodal plane; and at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$, the electric field exhibits an antinodal plane, but the magnetic field exhibits a nodal plane - thus, in the standing wave, the electric and magnetic fields are now 90 degrees out of phase

Standing waves can also be reflected between two conducting planes positioned along the X-axis at a second node - these planes form a cavity, and create a standing wave with frequency $f_n = \frac{c}{\lambda_n} = \frac{n\,c}{2L}$ - i.e., electromagnetic waves can exhibit a set of normal modes - this occurs in the center of a microwave oven

Partial reflection occurs via refraction: at an interface between material with different dielectric or magnetic properties, some of the electromagnetic wave is reflected, and some is transmitted